Answers

Exercise 1A. Page 24.

1. 5 **2.** $6x - 2$ **3.** $6x^2 - 2x$ **4.** -2

5. $\frac{1}{5}$ **6.** $-\frac{5}{x^2}$ **7.** $6x + \frac{6}{x^3}$ **8.** $\frac{5}{\sqrt{x}}$

9. $\frac{2}{\sqrt{x}}$ **10.** $-\frac{4}{x^{\frac{3}{2}}}$ **11.** $\frac{1}{3x^{\frac{2}{3}}}$ **12.** 5

13. $-\frac{1}{x^2}$ **14.** $70x$ **15.** $9x^2 + 4x - 3$

16. 2 **17.** $6x$ **18.** 6 **19.** $12x$

20. 4 **21.** $24x + 6$ **22.** $-\frac{1}{4x^{\frac{3}{2}}}$ **23.** $-\frac{2}{x^{\frac{3}{2}}}$

24. $\frac{2}{x^3}$ **25.** 0 **26.** $\frac{10}{x^3}$ **27.** $2 + \frac{24}{x^4}$

28. $3 + \frac{1}{x^2}$ **29.** $10x + \frac{4}{\sqrt{x}}$ **30.** $6\sqrt{x}$ **31.** $36x^2 + 24x$

32. $\frac{18}{x^5}$ **33.** $30x - \frac{6}{x^4}$ **34.** 4 **35.** 5

36. -8 **37.** -42

38. (a) $5 - 6x^2$ (b) -19 (c) $-12x$ (d) 24

39. $y = -20x - 20$ **40.** $y = -0{\cdot}5x + 6$ **41.** $y = x + 2$

42. (a) $(-3, 28)$ and $(1, 4)$ (b) $(0{\cdot}36, 8{\cdot}6)$

43. (a) $(3, 2{\cdot}25)$ (b) $(1, -1)$

44. $a = 5,\ b = 3,\ c = -1.$

Exercise 1B. Page 28.

1. $3x^2$ (as expected)

2. $2x + 7$ **3.** $2x + 4$ **4.** $6x + 13$ **5.** $6x + 7$

6. $120x + 17$ **7.** $24x + 28$ **8.** $3x^2 + 8x + 2$ **9.** $3x^2 + 10x - 3$

10. $3x^2 + 14x + 1$ **11.** $3x^2 - 20x + 8$ **12.** $6x^2 + 26x - 11$ **13.** $9x^2 - 10x$

14. $6x^2 + 14x - 17$ **15.** $9x^2 - 40x - 4$ **16.** 7 **17.** 4

18. 11 **19.** -5 **20.** $y = 13x - 22$ **21.** $y = 23x - 11$

22. $(-0{\cdot}5, -5)$ **23.** $(-2, 15),\ (4, 21)$ **24.** $(3, 0),\ (-1, 28)$

25. (a) $5x^{\frac{3}{2}} + \frac{3x^{\frac{1}{2}}}{2}$ (b) $5x^{\frac{3}{2}} + \frac{3x^{\frac{1}{2}}}{2}$

Exercise 1C. Page 30.

1. $2x$ (as expected)

2. $-nx^{-n-1}$ (as expected)

3. $\frac{6}{(x+3)^2}$ **4.** $-\frac{3}{(5x-1)^2}$ **5.** $-\frac{18}{(4x-3)^2}$ **6.** $\frac{7}{(1-2x)^2}$

7. $\frac{13}{(2x+3)^2}$ **8.** $-\frac{17}{(2x-3)^2}$ **9.** $\frac{17}{(5x+2)^2}$ **10.** $-\frac{1}{(2x-1)^2}$

11. $-\frac{6}{(3x+1)^2}$ **12.** $\frac{5(1-x^2)}{(x^2+1)^2}$ **13.** $\frac{2x(2-x^3)}{(x^3+1)^2}$ **14.** $\frac{9x(2-x^5)}{(x^5+3)^2}$

15. $-1{\cdot}5$ **16.** $-0{\cdot}625$ **17.** $y = -3{\cdot}5x + 27{\cdot}5$ **18.** $(1, 1)$ and $(1{\cdot}5, -2)$

19. (a) $\frac{3}{x^2}$ (b) $\frac{3}{x^2}$ (c) $\frac{3}{x^2}$

Exercise 1D. Page 35.

1. $7(4x+5)$ **2.** $12(2t+1)$ **3.** $40r(2r^2-1)$ **4.** $24(12x+5)$

5. $15(3x+2)^4$ **6.** $6x(x^2+2)^2$ **7.** $-\frac{8}{(8x-3)^2}$ **8.** $\frac{1}{\sqrt{2x+3}}$

9. $-\frac{3}{\sqrt{(6x+1)^3}}$ **10.** $-\frac{4(3x+1)}{(3x^2+2x+1)^3}$ **11.** $20(5x+2)^3$ **12.** $21(7x-3)^2$

13. $-9(2-3x)^2$ **14.** $14(4+7x)$ **15.** $18x(3x^2+5)^2$ **16.** $36x^2(2x^3+1)^5$

17. $-\frac{3}{(x+2)^4}$ **18.** $-\frac{2}{(2x+5)^2}$ **19.** $-\frac{1}{(x+2)^2}$ **20.** $-\frac{14}{(7x-3)^3}$

21. $3+10(2x+3)^4$ **22.** $\frac{1}{2\sqrt{x+1}}$ **23.** 50 **24.** 450

25. -48 **26.** -8 **27.** 0 **28.** 9

29. $-0{\cdot}125$ **30.** $-2{\cdot}5$ **31.** -1

Miscellaneous Exercise One. Page 37.

1. (a) $\{-3, -1, 5\}$ (b) $\{1, 9, 25, 49\}$ (c) $\{0, 1, 2\}$

2. (a) A, E (b) F (c) A, D, E (d) A, D, F

3. -14

4. (a) $10x$ (b) $10x$ (c) $30+50x$

5. (a) $2x-2$ (b) $20x+3$ (c) $8x+12$ (d) $9x^2+10x-12$

6. -56

7. $2x-3$

8. -8

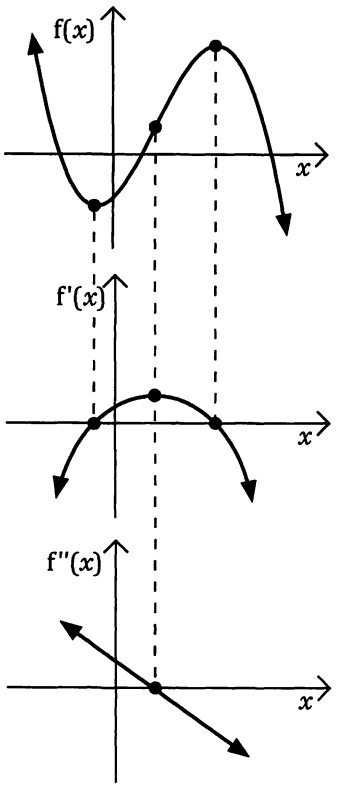
9. $16y = 5x-3$

10. $(0{\cdot}5, 2{\cdot}5)$, gradient $\frac{8}{3}$. $(3, 5)$, gradient $\frac{3}{8}$.

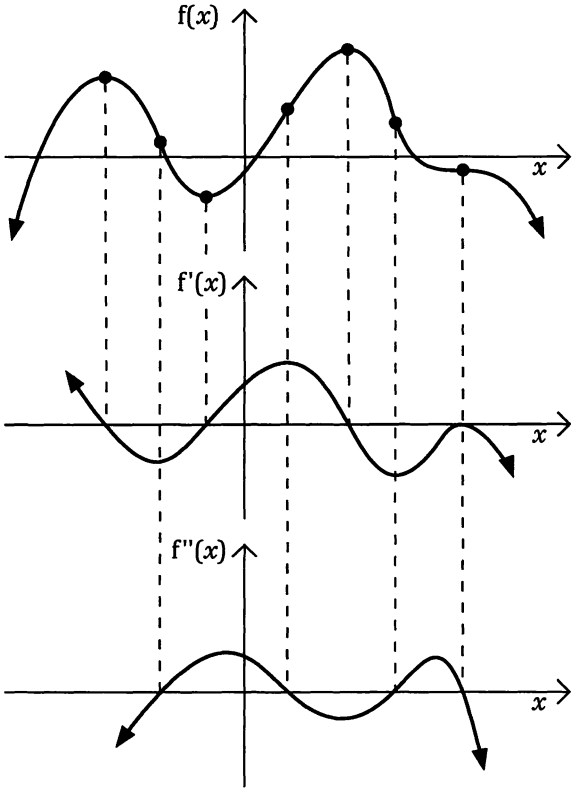
11. (a) $4x+7$ (b) $18x^2+38x-19$

Exercise 2A. Page 46.

1. (a)



(b)



2. Minimum at (6, 4).

3. Maximum at (4, 21).

4. Maximum at $(-\sqrt{3}, 6\sqrt{3})$, minimum at $(\sqrt{3}, -6\sqrt{3})$.

5. Maximum at (−1, 71), minimum at (7, −185).

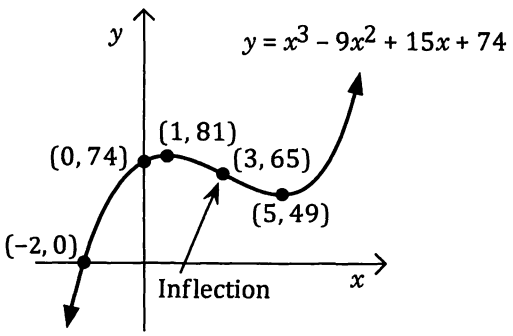
6. Minimum at (1, 2).

7. Maximum at (−5, −7), minimum at (−1, 1).

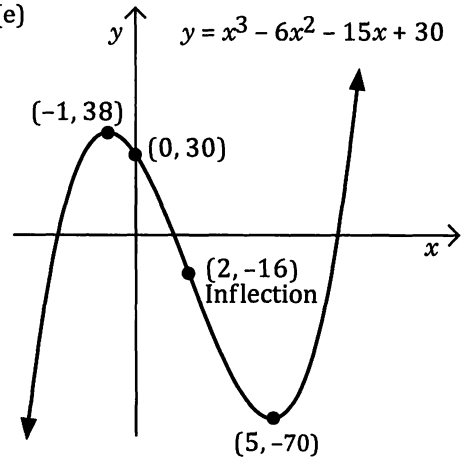
8. Maximum at $(-\sqrt{5}, -2\sqrt{5})$, minimum at $(\sqrt{5}, 2\sqrt{5})$.

9. Horizontal inflection at (0·5, 1).

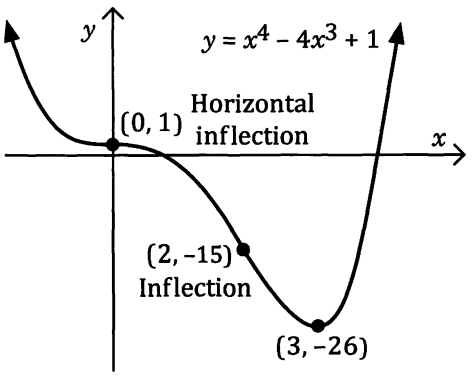
10.



11. (a) (0, 30)
(b) As $x \to \infty$, $y \to \infty$
(faster than x does).
As $x \to -\infty$, $y \to -\infty$
(faster than x does).
(c) Maximum at (–1, 38),
minimum at (5, –70).
(d) (2, –16)
(e)



12. (a) (0, 1)
(b) As $x \to \infty$, $y \to \infty$
(faster than x does).
As $x \to -\infty$, $y \to \infty$
(faster than $x \to -\infty$).
(c) Minimum at (3, –26),
(d) (0, 1) and (2, –15)
(e)



13. $\frac{dy}{dx} = 12(x+1)(x-3)^2$.
Min at (– 1, –256),
horizontal inflection at (3, 0).

14. (a) $2\frac{5}{27}$ (b) 4

15. (a) (0, 0) (b) Inflection at (0, 0), not horizontal inflection.

16. (a) (0, 0), (4, 256) (b) Horizontal inflection at (0, 0), Inflection (not horizontal) at (4, 256)

17. (a) (0, 0) (b) Not a point of inflection. [(0, 0) is a minimum point.]

18. $3x^2 - 3$, $6x$, $a = 0$. $(a, f(a))$, i.e. (0, –2), is a point of inflection (not horizontal).

Exercise 2B. Page 49.

1. $6a^2 + 3$ **2.** $1 - 10p + 6p^2$ **3.** $6(2t-1)^2$

4. $\frac{19}{(2x+5)^2}$ **5.** $18q^2 - 30q + 2$ **6.** (a) 6 cm^3/sec (b) 24 cm^3/sec (c) 54 cm^3/sec

7. (a) $30t^2 - 10t$ (b) (i) 20 insects/day (ii) 700 insects/day (iii) 2900 insects/day

8. (a) 15 m, 25 m/s (b) 275 m, 105 m/s (c) 4100 m, 405 m/s

9. (a) 5 (b) 6655 (c) $30(2t+1)^2$ bacteria/hour
(d) (i) 750 bacteria/hour (ii) 3630 bacteria/hour (iii) 13230 bacteria/hour

10. (a) 9 weeks (8·9) (b) $\frac{800}{(w+1)^2} - \frac{2500}{\sqrt{w}}$, –2300 \$/week, –1400 \$/week, –900 \$/week.

Exercise 2C. Page 52.

1. (a) positive (b) positive (c) negative (d) negative (e) positive (f) positive

2. (a) 26 m/s (b) 10 m/s^2

3. (a) 15 m/s (b) 12 m/s^2

4. $-0{\cdot}04$ m/s^2

5. 160 m/s^2

6. (a) 24 m/s^2 (b) 0 m/s^2 (c) 8 m/s^2 (d) $-\frac{1}{27}$ m/s^2 (e) 48 m/s^2 (f) 900 m/s^2

7. (a) -11 m/s (b) 2 m/s^2 (c) 8 (d) 3, 8

8. 9 m

Exercise 2D. Page 55.

1. For maximum profit $x = 50$. The max profit is then \$187 500.

2. 200 items should be made for a maximum profit of \$4 000 000.

3. For maximum profit $x = 70$. The max profit is then \$106 000 (nearest \$1000).

4. Maximum total weight of fruit per 100m^2 produced when N = 20.

5. (a) Maximum capacity is 2250 cm^3. (b) Maximum capacity is 3528 cm^3.

6. When $x = 200$ the average cost per unit has its minimum value of \$12.

8. The company should make 14 As and 204 Bs for a maximum profit of \$119 200.

9. $r = \frac{40}{\pi}$, $y = 40$.

10. The dimensions should be 80 m × 100 m with one of the 80 m sides fenced with the \$24 metre fencing. The minimum cost will then be \$6400.

11. $x = 8$ gives the greatest probability of recovery, this maximum being $0{\cdot}9375$.

12. Maximum P is $0{\cdot}72$ (i.e. 72%) when $t = 10$.

13. When $x = 5$ the average cost per unit is a minimum. The minimum average cost is \$675/unit.

14. Separation distance t hours later = $\sqrt{625 - 3000t + 10000t^2}$

The separation distance is a minimum after 9 minutes.

This minimum separation distance is 20 km.

Exercise 2E. Page 61.

1. $0{\cdot}24$, $f(4{\cdot}02) - f(4) = 0{\cdot}2404$

2. $0{\cdot}07$, $f(3{\cdot}01) - f(3) = 0{\cdot}0702$

3. $0{\cdot}15$

4. $1{\cdot}46$

5. $0{\cdot}18$

6. $-0{\cdot}004$ (i.e. a decrease)

7. $0{\cdot}1$

8. y increases by approximately 10%

9. y increases by approximately 6%

10. 2π cm^2

11. $6{\cdot}180$ cm, $0{\cdot}026$ cm

12. \$200

13. \$21

14. Surface area decreases by $0{\cdot}8\pi$ cm^2

15. $0{\cdot}8\%$

16. 2π cm^2

17. \$1 500

18. $(6 \pm 0{\cdot}011)$ cm

19. $\frac{\pi r^2}{20}$ m^3, $\frac{1}{2\pi}$ m (≈ 16 cm)

20. 120 cm^3

21. 3%

Miscellaneous Exercise Two. Page 63.

1. (a) $2\sqrt{2}$ (b) $4\sqrt{2}$ (c) $5\sqrt{2}$ (d) $3\sqrt{2}$ (e) $10\sqrt{2}$
(f) $4\sqrt{2}$ (g) $2\sqrt{2}$ (h) $4\sqrt{2}$ (i) $2\sqrt{2}$

2. $5x^4$ (as expected)

3. $5x^4$ (as expected)

4. $20x$

5. 5

6. 2

7. $6x^2 - 6x$

8. $15x^2 + 14x - 6$

9. 0

10. $12x + 1$

11. $18x^2 - 6x$

12. $8x + 20$

13. $14(2x-1)^6$

14. $-\dfrac{17}{(2x-3)^2}$

15. $-\dfrac{15x^2+6x+5}{(3x^2-1)^2}$

16. $30t$, 90

17. $6-\dfrac{4}{t^3}$, 5·5

18. $80(2t+3)^3$, 10000

19. $6t + 40 + 125\,t^{-\frac{3}{2}}$, 191

20. Stationary points occur for $x = 0$ (local minimum),
$x = \dfrac{25}{7}$ (local maximum),
$x = 12{\cdot}5$ (horizontal inflection).

21. $y = 3x + 9$

22. (a) 16250 (b) 25000

23. (a) $\dfrac{dy}{dx} = \dfrac{x^2-2x-2}{(x-1)^2}$
(b) $(1-\sqrt{3}, 2-2\sqrt{3})$, $(1+\sqrt{3}, 2+2\sqrt{3})$
(c) Maximum at $(1-\sqrt{3}, 2-2\sqrt{3})$, Minimum at $(1+\sqrt{3}, 2+2\sqrt{3})$.

24. $a = 5$, $b = -6$, $c = 14$, $y = -20x - 26$

25. y decreases by approximately 2%

26. Compare your answers with those of others in your class.

27. No we cannot conclude that (3, 1) is a point of inflection.
At all points of inflection, provided $f''(x)$ exists, $f''(x) = 0$ but a point where $f''(x) = 0$ does not have to be an inflection point. Consider $f(x) = (x-3)^4 + 1$ or $f(x) = (x-3)^4 + x - 2$ for example.

Exercise 3A. Page 67.

1. $\dfrac{x^7}{7} + c$

2. $\dfrac{x^4}{4} + c$

3. $2x^5 + c$

4. $\dfrac{7x^3}{3} + c$

5. $4x^2 + c$

6. $8x + c$

7. $\dfrac{2}{3}x^{\frac{3}{2}} + c$

8. $\dfrac{3}{4}x^{\frac{4}{3}} + c$

9. $\dfrac{2}{7}x^{\frac{7}{2}} + c$

10. $\dfrac{12}{5}x^{\frac{5}{2}} + c$

11. $8x^{\frac{1}{2}} + c$

12. $8\sqrt{x} + c$

13. $-\dfrac{10}{3x^3} + c$

14. $\dfrac{9}{x} + c$

15. $-32\sqrt{x} + c$

16. $2x^3 - 2x^2 + 3x + c$

17. $3x^4 + 3x + c$

18. $\dfrac{x^4}{4} + x^3 + x^2 + c$

19. $x + 2x^2 + 6x^3 + c$

20. $2x^{\frac{3}{2}} + 3x^2 + c$

21. $x^3 + 7x^2 + 8x + c$ **22.** $x^3 + 7x^2 + 8x + c$ **23.** $\frac{x^3}{3} + 2x^2 - 12x + c$

24. $3x^3 - 4x + c$ **25.** $3x^4 + 6x^2 + c$ **26.** $2x^2 + 5x + c$

27. $-\frac{2}{x} - \frac{1}{2x^2} + c$ **28.** $4x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + c$ **29.** $2\sqrt{x} - \frac{2}{5}x^{\frac{5}{2}} + c$

30. $2\sqrt{x} + x + c$ **31.** $y = 2x^3 + x - 5$ **32.** $y = 2x^2 - 3x + 2$

33. $A = t + \frac{6}{t} - 7$ **34.** $V = \frac{x^2}{2} + 2\sqrt{x} - 10$

35. (a) $\frac{2x^3}{5} + \frac{5}{6x} + \frac{5}{6}$ (b) $\frac{31}{15}$ (c) $-\frac{2}{5}$

Exercise 3B. Page 75.

1. $\frac{1}{12}(3x+2)^4 + c$ **2.** $\frac{1}{15}(3x+2)^5 + c$ **3.** $x^3 + x^2 + c$

4. $\frac{1}{25}(1+5x)^5 + c$ **5.** $-\frac{1}{20}(1-5x)^4 + c$ **6.** $(x^2+5)^5 + c$

7. $2(x^2-7)^5 + c$ **8.** $\frac{1}{2}x^2 + \frac{10}{3}x^3 + \frac{25}{4}x^4 + c$ **9.** $\frac{1}{6}(2x+1)^3 + c$

10. $x^4 + \frac{4}{3}x^3 + \frac{1}{2}x^2 + c$ **11.** $\frac{1}{20}(5x+1)^4 + c$ **12.** $-\frac{3(5-7x)^4}{4} + c$

13. $2(2x+1)^4 + c$ **14.** $3(3x-2)^5 + c$ **15.** $\frac{(x^2-x+3)^5}{5} + c$

16. $2(6x+1)^4 + c$ **17.** $\frac{1}{10}(5x+1)^4 + c$ **18.** $5(3x^2-6x+1)^5 + c$

19. $\frac{1}{3}(3x-1)^5 + c$ **20.** $\frac{1}{9}(9x+1)^3 + c$ **21.** $x^3 + 2x^2 + c$

22. $\frac{2}{9}(3x-1)^3 + c$ **23.** $\frac{1}{2}x^4 - \frac{4}{3}x^3 + x^2 + c$ **24.** $\frac{1}{3}x^3 - x + c$

25. $\frac{1}{4}(1+x)^4 + c$ **26.** $-\frac{1}{4}(1-x)^4 + c$ **27.** $\frac{1}{2}x^2 + \frac{1}{3}x^3 + c$

28. $\frac{x^4}{2} + \frac{4x^3}{3} + x^2 + c$ **29.** $2(1+x^2)^3 + c$ **30.** $\frac{(1+x^2)^7}{7} + c$

31. $3(1-2x)^4 + c$ **32.** $3(2x-1)^9 + c$ **33.** $-\frac{1}{2}(5-6x)^5 + c$

34. $-\frac{1}{8}(3-2x)^4 + c$ **35.** $\frac{1}{3}(2x-3)^9 + c$ **36.** $-\frac{1}{2}(5-6x)^4 + c$

37. $\frac{(x^2+x+3)^5}{5} + c$ **38.** $\frac{(5x^2+3)^8}{4} + c$ **39.** $-\frac{(x^2-x+3)^5}{5} + c$

40. $-\frac{1}{3(x+2)^3} + c$ **41.** $-\frac{5}{(x+1)} + c$ **42.** $-\frac{(x^2-2x+1)^4}{8} + c$

43. $-\frac{1}{(x+3)^2} + c$ **44.** $-\frac{3}{(x^2-3)^3} + c$ **45.** $\frac{1}{2-x} + c$

46. $\frac{1}{2(1-2x)} + c$ **47.** $\frac{5}{(3-2x)^2} + c$ **48.** $2(3x^2-x+1)^5 + c$

49. $\frac{1}{2(x-2)^2} + c$ **50.** $\frac{4}{1-3x} + c$ **51.** $\frac{2}{(1-5x)^2} + c$

52. $\frac{2}{9}(3x+2)^{\frac{3}{2}}+c$ **53.** $4(2x-5)^{\frac{3}{2}}+c$ **54.** $6\sqrt{1+2x}+c$

55. $x-\frac{1}{15}(1-5x)^3+c$ **56.** $3(3x-2)^{\frac{4}{3}}+c$ **57.** $x+\frac{1}{2}x^2-\frac{10}{3}x^3+\frac{25}{4}x^4+c$

58. $-\frac{2}{(2x-3)^3}+c$ **59.** $2(2x+1)^3+(3x-2)^3+c$ **60.** $\frac{2}{3}(x+3)^{\frac{3}{2}}+\frac{2}{3}(x+1)^{\frac{3}{2}}+c$

61. $10\sqrt{x^2+3x-1}+c$ **62.** $A=2(p+1)^3+5$ **63.** $y=2(2x+1)^5+23$

64. $f(x)=5-4(3-2x)^4$ **65.** $y=\frac{(5x^2-1)^3}{2}+8$ **66.** $v=8-\frac{25}{(t^2+1)^2}$

67. $x=\frac{5}{2t+1}+7$ **68.** (a) $y=3(2x-1)^4+2$ (b) 5 (c) -1 or 2

Exercise 3C. Page 77.

1. (a) 48 m/s^2 (b) 23 m

2. (a) 4 m/s^2 (b) 9 m/s (c) 26 m

3. (a) -10 m/s (b) 10 m/s (c) -30 m

4. 22·6 m, 4·88 m/s

5. 3·8 m

6. (a) 36 m/s (b) 145·8 m

7. 12 m

8. 700 m

9. $v=\frac{8}{5}(2t+1)^3+0{\cdot}8$, $x=\frac{(2t+1)^4}{5}+0{\cdot}8t+2.$

10. -6 m/s

11. (a) 6, 108 m (b) 18 m

12. (a) 1·8 km (b) 30 m/s

13. $v=u+at$, $s=ut+\frac{1}{2}at^2$

14. 75 m, 52·5 m/s

15. 52 m/s

Miscellaneous Exercise Three. Page 79.

1. $3x^2+6x+1$ **2.** $3x^2-10x-7$ **3.** $3x^2+4x+2$

4. $6x^2-6x+10$ **5.** $27x^2-6x-5$ **6.** $12x^2-38x-1$

7. (a) $10(2x-3)^4$ (b) 10 (c) $80(2x-3)^3$ (d) 80

8. a = 3·5, b = 4

9. $-8{\cdot}5$ m, $-13{\cdot}5$ m/s

10. (a) $(-2, 8)$

(b) $(-4, -44)$ and $(4, 44)$.

(c) $(1, 1)$ and $\left(-\frac{5}{3}, -\frac{5}{3}\right)$.

11. (a) 43·2km (b) 144km (c) 259·2km

12. \$900 per unit. It will cost approx \$900 to produce the 101st item.

13. 15 m by 20 m with one of the 15 m sides having the \$15 fencing.

14. (a) (i) \$10 (ii) \$53 (iii) \$1000 (iv) \$19 000 (b) $\dfrac{100\,000}{(100-p)^2}$

15. (a) (i) \$5560 (ii) \$9400 (b) (i) \$347·50 (ii) \$23·50

(c) $5+\dfrac{60}{\sqrt{x}}$ (d) (i) \$20 (ii) \$8

(e) C(17) – C(16) ≈ \$19·77, C(401) – C(400) ≈ \$8·00

16. Maximum at (–3, –19), minimum at (–1·5, – 28), maximum at (–1, –27), minimum at (0·5, 324).

Exercise 4A. Page 84.

1. (a) Approximately 0·2 units^2.
   (b) Underestimate: 0·14979. Overestimate: 0·24354. Mean: 0·196665. I.e. approx 0·197.
2. (a) Approximately 1·7 units^2.
   (b) Underestimate: 1·36. Overestimate: 1·96. Mean: 1·66.
3. (a) Approximately 6·6 units^2.
   (b) Underestimate: 5·88. Overestimate: 7·48. Mean: 6·68.

Exercise 4B. Page 90.

1. (a) 76 units^2 (b) 76 units^2 **2.** (a) 27 units^2 (b) 27 units^2

3. (a) 14 **4.** (a) 2 **5.** $\frac{2}{3}$ **6.** $4\frac{2}{3}$

7. 40 **8.** 10 **9.** 0 **10.** 19

11. 39 **12.** 12 **13.** 76·5 **14.** $2\frac{2}{3}$

15. 42·2 **16.** 671 **17.** 14 **18.** 0

19. $3\frac{11}{15}$ **20.** (a) $\frac{1}{3}$ (b) $8\frac{2}{3}$ (c) 9

21. (a) $10\frac{2}{3}$ (b) $-2\frac{1}{3}$ (c) $8\frac{1}{3}$ **22.** (a) 34 (b) –34

23. (a) 9 (b) 27 (c) 36 **24.** 6π **25.** $18-4\sqrt{2}$

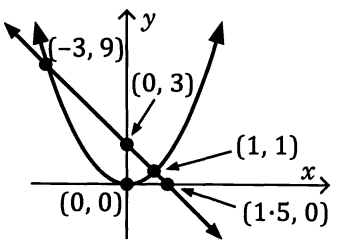
Exercise 4C. Page 98.

1. a, e, f **2.** (a) No (b) No (c) Yes (d) No (e) Yes (f) Yes (g) Yes **3.** 6 units^2

4. 49·6 units^2 **5.** $6\frac{2}{3}$ units^2 **6.** $21\frac{1}{3}$ units^2 **7.** 0·75 units^2 **8.** 2 units^2

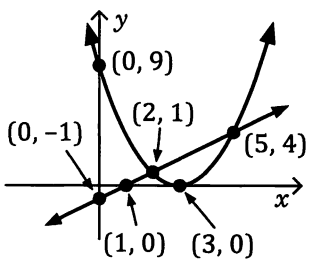
9. 2 units^2 **10.** 3·5 units^2 **11.** 0·5 units^2 **12.** 3 units^2 **13.** $\frac{8\sqrt{2}}{3}$ units^2

14.



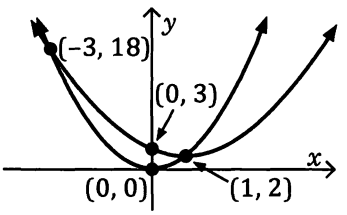
$10\frac{2}{3}$ units^2

15.



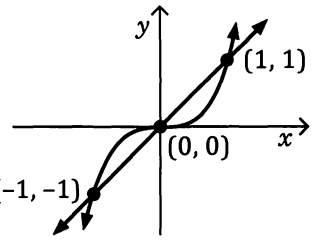
4·5 units^2

16.



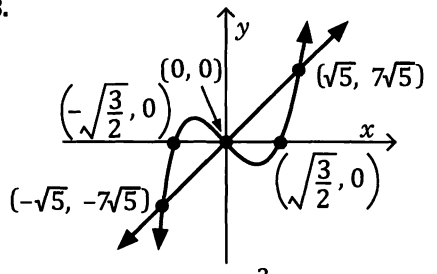
$10\frac{2}{3}$ units^2

17.



0·5 units^2

18.



25 units^2

19. \$3000

Exercise 4D. Page 102.

1. (a) \$3 000 (b) \$39 000

2. \$2500

3. (a) \$259 (b) \$268

4. (a) $\int_5^8 40(25 - t)\,dt$ (b) 2 220

5. (a) $\int_5^{10} \frac{t^{0\cdot1}}{2}\,dt$ (b) 3·1

6. 48·5 million

7. (a) (i) $\int_0^{10} (20 - 0{\cdot}15t^2)\,dt$ (ii) 150 kL (b) (i) $\int_0^1 (20 - 0{\cdot}15t^2)\,dt$ (ii) 20 kL

(c) (i) $\int_9^{10} (20 - 0{\cdot}15t^2)\,dt$ (ii) 6 kL

8. (a) 2880 (b) 620

9. (a) 400 (b) 560 (c) 100 (d) 130

Miscellaneous Exercise Four. Page 104.

1. A: $y = 4$ B: $x = -5$ C: $y = 0{\cdot}5x + 2$ D: $y = x + 3$
E: $y = 2x - 4$ F: $y = -x - 1$ G: $y = x - 6$ H: $y = -2x - 10$

2. (a) $2 - 9x^2$ (b) −223 (c) $-18x$ (d) 90

3. (a) 8 (b) $\frac{14}{3}$

4. $2x + 2$

5. $-2x - 2$

6. $4x + 11$

7. $8 - 8x$

8. $2(x + 1)(3x + 8)$

9. $(2x + 5)^2(40x + 61)$

10. (a) $y = -12x - 8$ (b) $y = -4x + 2$ (c) $y = 12x - 23$ (d) $y = -5x + 27$

11. (0, 3), (1, 0)

12. (−1·5, 0), (2, 0), (5, 0)

13. (a) $5x^4 + c$ (b) $4x^{\frac{3}{2}} + c$ (c) $\frac{(x+3)^5}{5} + c$

(d) $\frac{(2x+3)^5}{10} + c$ (e) $4(1 + x^3)^5 + c$ (f) $x + \frac{2x^3}{3} + \frac{x^5}{5} + c$

14. (a) $7{\cdot}5x - 6000$ (b) 800 (c) \$18 per unit, \$25·50 per unit, \$7·50 per unit

15. 53 m, 45 m/s

16. (a) $A = \frac{(2p-1)^4}{8} + \frac{3}{8}$ (b) $A = (p^2 - 1)^4 - 36$

17. (a) 8 units^2 (b) 6 units^2

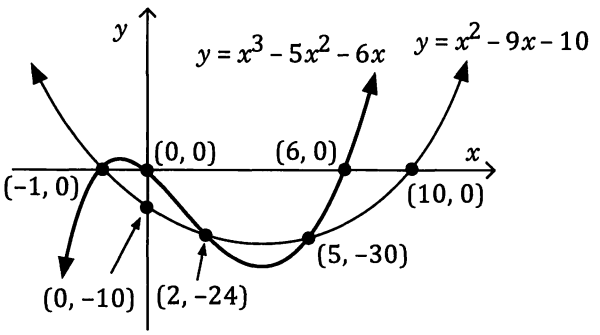
18. (a) $2x^{\frac{3}{2}} + 2\sqrt{x} + c$

19. (a) $y = x^3 - 5x^2 - 6x$ cuts the axes at (−1, 0), (0, 0) and (6, 0).

$y = x^2 - 9x - 10$ cuts the axes at (−1, 0), (0, −10) and (10, 0).

(b) a = 0, b = −24, c = −30.

(c)



(d) 40·5 units^2

Exercise 5A. Page 108.

1. (a) $4t^3 + 3t^2 + c$ (b) $4x^3 + 3x^2 - 7$ (c) $12x^2 + 6x$

2. (a) $t + \frac{1}{t} + c$ (b) $x + \frac{1}{x} - \frac{10}{3}$ (c) $1 - \frac{1}{x^2}$

3. (a) $\frac{(t^2+3)^5}{5} + c$ (b) $\frac{(x^2+3)^5}{5} - \frac{16807}{5}$ (c) $2x(x^2 + 3)^4$

4. $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$

5. $4x$ **6.** $5x^2$ **7.** $2x^3$

8. $\frac{2x}{5-x}$ **9.** $(x + 3)^4$ **10.** $16x(x^2 + 3)^4$

Exercise 5B. Page 113.

1. $2x + 3x^2$ **2.** $x^4 + 5$

3. (a) $x^2 + c$ (b) $6x^3 - 4x^2 + 2x + c$

4. (a) 1 (b) −2 (c) 8 (d) −2

5. (a) −2 (b) 3 (c) 5 (d) 9

6. (a) π (b) $4 - \pi$ (c) 0, 4 and 8 (d) $4 < a < 8$

7. $\frac{5}{2}x^{\frac{3}{2}} + (1 + 3x^2)^4$

Miscellaneous Exercise Five. Page 115.

1. $6x^2 + \frac{1}{2\sqrt{x}}$ **2.** $2x + 2$ **3.** $6(3x - 1)$

4. $15(3x - 1)^4$ **5.** $40x^3 - 6x^2 - 15$ **6.** $(2x - 3)^2 (40x - 21)$

7. $-\frac{5}{(x-1)^2}$ **8.** $\frac{5}{(2x+3)^2}$ **9.** -2

10. Max at $(-\sqrt{6}, -2\sqrt{6})$, min at $(\sqrt{6}, 2\sqrt{6})$.

11. (a) The function $f(x)$ has a value of 3 when $x = 0$.
The function $f(x)$ has a value of 28 when $x = 5$.
The average rate of change of $f(x)$, from $x = 0$ to $x = 5$, is 5 units per unit change in x.
The instantaneous rate of change of $f(x)$ when $x = 1$ is 2.

(b) $f(x) = x^2 + 3$

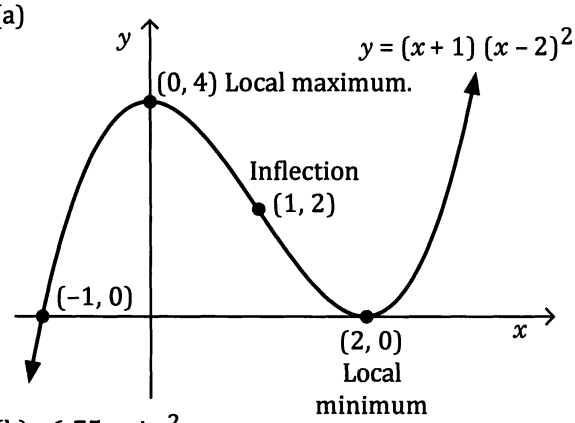
(c) Many possible cubics but all must have $d = 3$, $25a + 5b + c = 5$ and $3a + 2b + c = 2$.
One possibility is $f(x) = 3x^3 - 21x^2 + 35x + 3$.

12. $y = 5(2x + 1)^5 + 2$

13. $f(x) = 3(2x - 1)^4 + 2x + 1$

14. $30 - 0{\cdot}04x$, \$26 per unit, \$26

15. (a)



(b) $6{\cdot}75$ units^2

16. Underestimate: 3·4075.
Overestimate: 4·1075.
Mean: 3·7575.

17. (524 ± 31) cm^3

Exercise 6A. Page 120.

1. (a) \$1 822·12 (b) \$3 320·12 (c) \$20 085·54

2. \$12 500

3. ~26 g

4. (a) ~2 000 000 (b) ~1 500 000 (c) ~1 100 000 (d) ~800 000

5. (a) 35·8 m/sec (1dp) (b) 69·4 m/sec (1dp) (c) 74·6 m/sec (1dp)

6. (a) 0 (b) 27·73 (c) 73·78

7. ~18

8. (a) \$36 112·55 (b) ~20 years (t = 19·98 to 2 dp)

Exercise 6B. Page 126.

1. e^x **2.** $7e^x$ **3.** $3e^x$

4. $6e^x$ **5.** $9e^x$ **6.** $-8e^x$

7. $5e^{5x}$ **8.** $7e^{7x}$ **9.** $-2e^{-2x}$

10. $15e^{3x}$ **11.** $2e^{0{\cdot}5x}$ **12.** $e^{-0{\cdot}5x}$

13. $6e^x + 6x^2 + 6x$ **14.** $2e^x + \frac{1}{2\sqrt{x}}$ **15.** $5e^{5x} + 2e^{2x}$

16. $8e^{4x}$ **17.** $6e^{3x} + 6e^{2x}$ **18.** $15e^{3x} + 4x^3$

19. $3e^{3x-1}$ **20.** $2xe^{x^2+3}$ **21.** $5e^{5x-1}$

22. $(6x+2)e^{3x^2+2x-1}$ **23.** $3x^2e^{x^3}$ **24.** $e^{2x}(1+2x)$

25. $x^2e^x(3+x)$ **26.** $\frac{e^x(1+2x)}{2\sqrt{x}}$ **27.** $\frac{e^x(x-1)}{2x^2}$

28. $e^x(1+2x)^2(7+2x)$ **29.** $-e^x(1-2x)^4(2x+9)$ **30.** $-\frac{3}{e^{3x}}$

31. $2(e^2+1)$ **32.** $2e$ **33.** $y = 10x + 5$

34. (a) \$8·67 per yr (b) \$17·80 per yr (c) \$39·62 per yr (d) \$196·26 per yr

35. (a) 100 (b) 61 tonnes (c) 8·19 tonnes/wk (d) 6·07 tonnes/wk
(e) 4·49 tonnes/wk

Exercise 6C. Page 130.

1. (a) ~609 (b) ~90400

2. (a) ~2210 (b) ~3297

3. (a) ~159 (b) ~318

4. (a) ~7358 (b) ~2707

5. (a) $\sim 7{\cdot}309 \times 10^7$ (b) $\sim 1{\cdot}085 \times 10^{10}$

6. (a) ~2050 (b) ~2570

7. (a) ~340 million (b) ~1120 million

8. (a) ~320 million (b) ~870 million

9. ~272 g **10.** \$78213·16 **11.** \$1814·36 **12.** \$118·73

13. \$2384·77

14. (a) ~7800 (b) ~6100

15. (a) 0·02 (b) ~2046

16. (a) ~5·2 million (b) ~18 million

17. (a) ~5·8 (b) ~6·3 (c) ~0·58 hrs (d) ~1·16 hrs

18. ~740

19. ~3·3

Exercise 6D. Page 133.

1. $2e^{3x} + c$ **2.** $3e^{2x} + c$ **3.** $\frac{1}{5}e^{5x} + c$ **4.** $\frac{1}{3}e^{9x} + c$

5. $\frac{5}{3}e^{3x} + c$ **6.** $-\frac{5}{e^x} + c$ **7.** $8\sqrt{e^x} + c$ **8.** $-\frac{1}{2e^{2x}} + c$

9. $2e^{2x} + x^2 + c$ **10.** $\frac{1}{3}e^{3x} + \frac{1}{2}e^{2x} + c$ **11.** $-\frac{3}{2e^{2x}} + c$ **12.** $-\frac{2}{e^{2x}} + \frac{e^{2x}}{8} + c$

13. $e^{x^2} + c$ **14.** $3e^{2x+1} + c$ **15.** $4e^{x^2+5} + c$ **16.** $5(e^2 - 1)$

17. $\frac{e^5 - 1}{5}$ **18.** $2e^4 - e^2 - e$ **19.** $3 + e^4$ **20.** $e - 1$

21. $12e + 4$

22. (a) $\frac{1 + 5e^{2t}}{2}$ (b) $\frac{1 + 5e}{2}$

23. (a) $2x^3 - 4e^{3x} + 5$ (b) $21 - 4e^6$

24. (a) 19·1 units^2 (b) $(e^3 - 3e + 1)$ units^2

Miscellaneous Exercise Six. Page 134.

1. $y = 13x - 8$

2. (a) $5(x+2)^4$ (b) $10(2x+1)^4$ (c) $\dfrac{10}{(x+5)^2}$

(d) $\dfrac{26}{(x+5)^2}$ (e) $12x^2 - e^x$ (f) $5e^{5x} + 5$

3. $a = 0{\cdot}4,\ b = 50$

4. (a) $0{\cdot}1$ m/s^2 (b) $0{\cdot}1e^2$ m/s^2 ($\approx 0{\cdot}739$ m/s^2)

(c) $(10e + 2)$ m ($\approx 29{\cdot}18$ m)

6. $(-5, 0)$ gradient 35, $(0, 0)$ gradient -10, $(2, 0)$ gradient 14.

Total area enclosed by curve and x-axis is $101{\cdot}75$ units^2 ($= 93{\cdot}75$ units$^2 + 8$ units^2).

7. $a = -1,\ b = 4$

8. (a) 48 (b) $0{\cdot}5$ (c) $e - 1$

(d) $3e^2 - 3$ (e) 15 (f) $\dfrac{5}{3}$

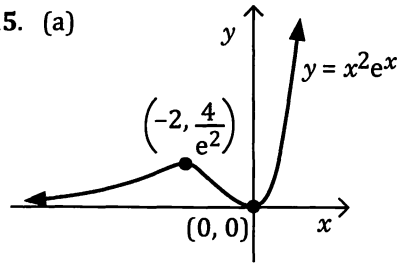
9. Answers to be checked with calculator.

10. (a) 7·5km (b) 27·5km (c) 31·5km

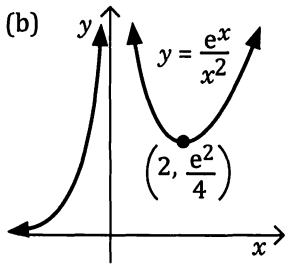
11. 35·2 m/s (rounded to 1 dp), $\dfrac{200}{3}$ m/s. **12.** 16π cm^2 **13.** (a) \$745·91 (b) \$1660·06

14. (a) 21·1°C/min (b) 11·6°C/min (c) 0·3°C/min

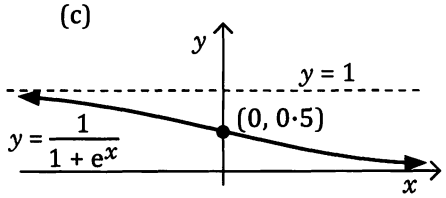
15. (a)



(b)



(c)



16. 613 cm^2

Exercise 7A. Page 148.

1. $5x^4 - 2x$ **2.** $3x^2$ **3.** $\sin x$

4. $\cos x + \sin x$ **5.** $-\sin x - \cos x$ **6.** $1 - \dfrac{1}{\cos^2 x}$ (i.e. $-\tan^2 x$)

7. $4x - 1$ **8.** $10x - 75x^2$ **9.** $6\cos x$

10. $-4\sin x$ **11.** $\sin x + x\cos x$ **12.** $2x\cos x - x^2\sin x$

13. $-\dfrac{3x^2+1}{(3x^2-1)^2}$ **14.** $-\dfrac{4x}{(x^2-1)^2}$ **15.** $-\dfrac{x\sin x + \cos x}{x^2}$

16. $\dfrac{x\cos x - \sin x}{x^2}$ **17.** $\dfrac{\sin x - x\cos x}{\sin^2 x}$ **18.** $\dfrac{\cos x + x\sin x}{\cos^2 x}$

19. $12x(x^2+1)$ **20.** $\dfrac{x}{\sqrt{x^2-1}}$ **21.** $6\cos 6x$

22. $-2\sin(2x+3)$ **23.** $2\sin x\cos x$ **24.** $3\sin^2 x\cos x$

25. $-5\cos^4 x\sin x$ **26.** $-3\sin 3x$ **27.** $3\cos(3x-7)$

28. $-2\sin(2x+5)$ **29.** $3\sin x$ **30.** $3 - 2\sin x$

31. $2\cos 2x$ **32.** $2x + \sin x$ **33.** $\dfrac{x\cos x - 2\sin x - 2}{x^3}$

34. $3\cos x + 2\sin x$ **35.** $-3\sin 3x$ **36.** $-9\sin 9x$

37. $-6\sin 2x$ **38.** $15\cos 3x$ **39.** $6\cos 3x - 6\sin 2x$

40. $5\sin^4 x\cos x$ **41.** $-10\cos x\sin x$ **42.** $\dfrac{\cos x}{2\sqrt{\sin x}}$

43. $7\cos 7x$ **44.** $8\cos 8x$ **45.** $4\cos 4x - 4\sin 4x$

46. $6\cos(3x-1)$ **47.** $-16\sin(4x+3)$ **48.** $6\sin^2 x\cos x$

49. $-6\cos x\sin x$ **50.** $\cos x - x\sin x$ **51.** $2x\cos x - x^2\sin x$

52. $2\sin x + 2x\cos x$ **53.** $\dfrac{2}{\cos^2 x}$ **54.** $\dfrac{2}{\cos^2 x}$

55. $\dfrac{\sqrt{3}}{2}$ **56.** $-\sqrt{3}$ **57.** 2

58. 0 **59.** $-\sin x$ **60.** $-25\cos 5x$

61. $-12\sin 2x$ **62.** $-\sin x - \cos x$ **63.** $y = x$

64. $y = x + 3$ **65.** (a) 1 (b) $-2\sqrt{3}$ **66.** $\dfrac{\pi}{180}\cos x^\circ$

67. 200 cm^2, $10\sqrt{2}\text{ cm}$

68. $4\cos(0{\cdot}1t)\text{ cm}^2/\text{s}$ (a) $3{\cdot}98\text{ cm}^2/\text{s}$ (b) $3{\cdot}51\text{ cm}^2/\text{s}$ (c) $2{\cdot}16\text{ cm}^2/\text{s}$ (d) $-1{\cdot}66\text{ cm}^2/\text{s}$

69. (a) $5, \dfrac{\pi}{6}$ (b) $\dfrac{\pi}{18}, \dfrac{5\pi}{18}, \dfrac{13\pi}{18}$ (c) $-3{\cdot}4$ (d) -9 **70.** $0{\cdot}6435,\ 5$

Exercise 7B. Page 154.

1. $5\sin x + c$ **2.** $-2\cos x + c$ **3.** $10\cos x + c$

4. $-2\sin x + c$ **5.** $3\sin 2x + c$ **6.** $\dfrac{1}{3}\sin 6x + c$

7. $-3\cos 4x + c$ **8.** $\dfrac{1}{3}\cos 3x + c$ **9.** $-\dfrac{4}{5}\sin 10x + c$

10. $-2\cos\dfrac{x}{2} + c$ **11.** $\dfrac{2}{3}\sin\dfrac{3x}{2} + c$ **12.** $9\cos\dfrac{2x}{3} + c$

13. $-3\cos(2x+3) + c$ **14.** $\dfrac{3}{2}\sin(2x-3) + c$

15. $\dfrac{1}{2}\sin\left(2x + \dfrac{2\pi}{3}\right) + c$ **16.** $\cos(-x) + c$, i.e. $\cos x + c$

17. $4\tan x + c$ **18.** $3\sin 2x - 2\cos 3x + c$

19. $\dfrac{1}{8}\sin 8x + 2\cos 2x + c$ **20.** $x^2 + 4\sin x + 3\sin 2x + c$

21. $3x + 2x^2 - 2x^3 + 2\sin 5x + \dfrac{1}{2}\cos 4x + c$

22. $-\dfrac{1}{4}\cos^4 x + c$ **23.** $-5\cos^6 x + c$ **24.** $-\dfrac{1}{7}\cos 7x + c$

25. $-\dfrac{1}{2}\cos 2x + c$ **26.** $\dfrac{1}{7}\sin 7x + c$ **27.** $\dfrac{1}{4}\sin 4x + c$

28. 1 **29.** 1 **30.** $2 - \sqrt{2}$

31. (a) $1 - \dfrac{1}{\sqrt{2}}$ (b) $\dfrac{1}{\sqrt{2}} - 1$ **32.** 2 units^2 **33.** (a) $0{\cdot}5\text{ units}^2$ (b) $2{\cdot}5\text{ units}^2$

34. (Could use calculus to determine the greatest speed and the least distance but easier to use the facts that $-1 \le \sin 2t \le 1$ and $-1 \le \cos 2t \le 1$.)
(a) 2 m/s (b) $(5 + \sin 2t)$ metres (c) 4 metres (d) $-4\sin 2t\text{ m/s}^2$

35. (a) $0{\cdot}25\text{ units}^2$ (b) $2{\cdot}5\text{ units}^2$

36. (a) A $(\pi/2, 0)$, B $(\pi, 0)$, C $(3\pi/2, 0)$ (b) 6 units^2

Miscellaneous Exercise Seven. Page 156.

1. e^x **2.** $2e^x$ **3.** $8e^x$

4. $e^x + \cos x$ **5.** $-\sin x\, e^{\cos x}$ **6.** $2\cos 2x\, e^{\sin 2x}$

7. $2\cos x\, e^{2\sin x}$ **8.** $e^x - \frac{1}{x^2}$ **9.** $\frac{2}{\sqrt{x}} + 3e^{3x}$

10. $\frac{e^x(2x+1)}{2\sqrt{x}}$ **11.** $e^x(\cos x + \sin x)$ **12.** $e^x(\cos 2x - 2\sin 2x)$

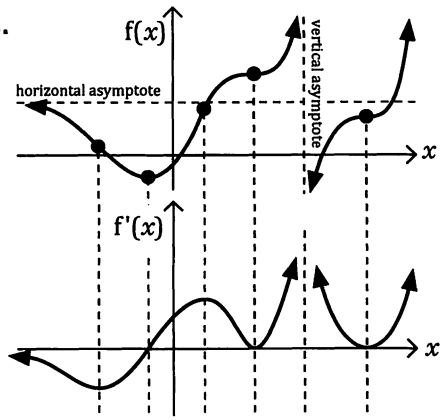
13. $e^x \sin x\ (\sin x + 2\cos x)$ **14.** $6x\, e^{3x^2+2}$ **15.** $(2x + \cos x)\, e^{x^2 + \sin x}$

16. $6(2r+3)^2$ **17.** (a) $2(e^4 - 1)$ (b) $\frac{3}{10}$ (c) 6

18. $-\frac{5\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$ **19.** $-e^{-\pi}$ **20.** $0{\cdot}5x^{-0{\cdot}5}$

21. A is decreasing as t increases.
(a) −245 619 tonnes per year (nearest 1 tonne/yr).
(b) −181 959 tonnes per year (nearest 1 tonne/yr).
(c) −110 364 tonnes per year (nearest 1 tonne/yr).

22.



23. (a) 10 m/s
(b) $-2{\cdot}5$ m/s^2
(c) 145 m
(d) 85 m
(e) 115 m
(f) 75 m

24. (a) $V = \frac{4}{3}\pi(100 - 3x)^3$ for $V \geq 0$.
(b) On the 7th day. ($x \approx 6{\cdot}88$).
(c) Decreasing at $12\pi(100 - 3x)^2$ m^3/day (d) ~270 000 m^3/day
(e) $8\pi(1 + x)(100 - 2x - x^2)^2$ m^3/day, ~640 000 m^3/day

25. (a) 0·66 m/s (b) 0·27 m/s^2 (c) 0·05 m/s^2

26. (a) $(1 + \frac{\sqrt{3}}{2} - \frac{5\pi}{24})$ units^2 (b) $(2 + \sqrt{3} - \frac{5\pi}{12})$ units^2

Exercise 8A. Page 167.

1. (a) Continuous (b) Discrete (c) Continuous (d) Discrete
(e) Discrete (f) Continuous (g) Continuous

2. No **3.** No **4.** No **5.** Yes

6. 0·4 **7.** 0·05 **8.** 0·1 **9.** −0·8

10.

x	0	1	2
$P(X = x)$	0·25	0·5	0·25

11. (a) 0·2 (b) 0·8 (c) 0·2 (d) 0·5 (e) 0·25 (f) 0·75

12. (a) 0·4 (b) 0·4 (c) 0·5 (d) 0·5 (e) 0·25 (f) 0·5

13.

x	0	1	2	3	4	5	6	7	8	9	10
$P(X \leq x)$	0·005	0·015	0·055	0·175	0·375	0·625	0·825	0·945	0·985	0·995	1

14.

x	0	1	2	3	4	5
$P(X = x)$	0·04	0·16	0·3	0·3	0·16	0·04

15.

	0	1	2
$P(X = x)$	0·16	0·48	0·36

16.

x	0	1	2	3
$P(X = x)$	$\frac{8}{27}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{1}{27}$

17.

x	1	2	3
$P(X = x)$	0·3	0·6	0·1

18.

x	1	2	3	4	5
$P(X = x)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$

(a) $\frac{2}{5}$ (b) $\frac{1}{15}$ (c) $\frac{4}{5}$

19.

x	1	2	3	4
$P(X = x)$	0·4	0·3	0·2	0·1

(a) 0·4 (b) 0·7 (c) 0·6

20. (a) 0·3 (b) 0·12 (c) 0·24 (d) 0·25 (e) 0·48 (f) 0·012 (g) 0·072 (h) 0·117 (i) 0·1

21.

x	0	1	2	3	4
$P(X = x)$	0·64696	0·30808	0·04299	0·00195	0·00002

Exercise 8B. Page 175.

1. $k = 0·1$, $E(X) = 2·7$

2. $k = 0·2$, $E(X) = 17$

3. $k = 0·05$, $E(X) = 5·85$

4. $k = 0·2$, $E(X) = 11·6$

5. $p = 0·15$, $q = 0·25$, $\text{Var}(X) = 1·91$

6. $p = \frac{5}{18}$, $q = \frac{2}{9}$

7. (a) 15 (b) 1·5 (c) 26 (d) 4·5

8. (a) $E(X) = 26$, $\text{Var}(X) = 184$ (b) 29 (c) 52
(d) 55 (e) 184 (f) 736 (g) 736

9. $E(X) = 3$, $\text{Var}(X) = 2$.

10.

x ($ paid out)	0	15	30
$P(X = x)$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

For the desired long term average profit per game they should charge $8 per play.

11. To at least break even in the long run (or be very close to break even) the organisers need to charge at least $2·50 per game.

12. (a) $E(X) = 4·5$ (b) $E(Y) = 25·5$ (c) $E(Z) = \frac{761}{2240}$

13. (a) Would expect approximately 12% of the games to result in prize of more than $6.
(b) Would expect 40% of the scores to exceed a total score of 4.
(c) Organisers should expect to be "up" by approximately $100 after 100 plays of the game.

14. Scheme ① has expected fortnightly earnings (i.e. long term average fortnightly earnings) of $962·50.
Scheme ② has expected fortnightly earnings (i.e. long term average fortnightly earnings) of $878·75.
Based on expected earnings he should prefer scheme ①.

15. (a) $1340 (b) $1270
(c) Compare your answers and justifications with those of others in your class but note that consideration of the probability distributions should suggest that it is perhaps not as straightforward as simply choosing the scheme with the higher expected return. Hence your response should discuss more than just choosing the higher expected return.

Miscellaneous Exercise Eight. Page 178.

1. (a) ~200 (b) ~298 (c) ~444 (d) As $t \to \infty, N \to 100\,000$.

2. (a) $-\frac{6}{x^2}$ (b) $-\frac{3}{\sqrt{x^3}}$ (c) $10x - e^x$ (d) $6xe^{3x^2}$

(e) $6xe^{3x^2+1}$ (f) $4(2x+1)^4(6x-7)$ (g) $10\cos x$ (h) $10\cos 10x$

3. $3x^2 - 5$

4. A: X is not a uniform discrete random variable because whilst the possible values of X are discrete values (0, 1 and 2), the probabilities $P(X = x)$ are not the same for all values of x. In this case $P(X=0) = \frac{25}{36}$, $P(X=1) = \frac{5}{18}$, $P(X=2) = \frac{1}{36}$.

B: X is a uniform discrete random variable because the possible values of X are discrete values (1, 2, 3, 4, 5, 6) and in each case the probability of occurrence is the same, i.e $\frac{1}{6}$.

C: X is not a uniform discrete random variable because the variable involved is continuous, not discrete.

5. (−1·5, −4·5) grad 4·2

6. $4e^2$

7. $3y = x + 24$

8. (a) 64 (b) 4 (c) 17

9. 1 unit^2

10. (a) 75 m/s (b) (i) 5·09 m/s^2 (ii) 0·72 m/s^2

11. $f(x) = (3-x)^5 + x^3 - 3x^2 - 2$

12. $5x^4$ (as expected).

13. (1, 0·5e)

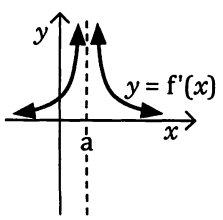
14. $-\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$

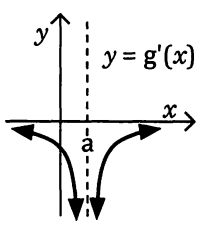
15. (a) $2x^2 + c$ (b) $3e^{2x} + c$ (c) $x^2 + e^x + c$ (d) $x^2e^x + c$

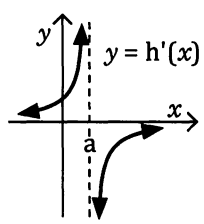
16. (a) $\frac{1}{12}$ (b) $\frac{1}{3}$ (c) $\frac{5}{12}$ (d) $\frac{3}{4}$ (e) 0 (f) $\frac{4}{9}$

(g) $\frac{41}{12}$ (h) 1·32 (Note: The exact value, not requested, would be $\frac{\sqrt{251}}{12}$.)

17.







18. (a) $\frac{5\pi - 6\sqrt{3} - 12}{12}$ (b) $\frac{12 + 6\sqrt{3} - 5\pi}{12}$ (c) $\frac{6\sqrt{3} - \pi - 4}{4}$ units^2

Exercise 9A. Page 190.

1. Mean = 0·6, variance = 0·24.

2. P(success) = 0·1 is graph B, P(success) = 0·5 is graph A, P(success) = 0·8 is graph C.

3. P(success) = 0·5 is graph C, P(success) = 0·7 is graph B, P(success) = 0·9 is graph A.

4. Mean = 3, standard deviation = 1·5 **5.** n = 24, p = 0·4

6. (a) a = 0·2076, b = 0·0865 (b) $\mu = 2, \sigma = 0{\cdot}5\sqrt{6}$ i.e. 1·225 rounded to 3 dp

(c) $P(\mu - \sigma \le X \le \mu + \sigma) = 0{\cdot}786$ rounded to 3 dp

(For numbers 7 to 13 answers are given rounded to 4 dp unless question requests otherwise.)

7. (a) 0·0605 (b) 0·0101 (c) 0·0705 (d) 0·9295

8. (a) 0·3025 (b) 0·1176 (c) 0·4202 (d) 0·5798

9. (a) 0·2605 (b) 0·0004 **10.** 0·1715

11. (a) 0·2001 (b) 0·2335 (c) 0·1493 (d) 0·3828

12. (a) 0·202 (b) 0·010 (c) 0·037 **13.** 0·1792

Exercise 9B. Page 194.

Unless requested otherwise answers tend to be given rounded to 4 dp or, if answer may have been obtained using previous 4 dp answers, then answer might be given rounded to 3 dp.

1. (a) 0·0459 (b) 0·0011 (c) 0·9999 (d) 0·9999
2. (a) 0·1171 (b) 0·1244 (c) 0·8744 (d) 0·8744
3. (a) 0·1612 (b) 0·4826 (c) 0·334
4. (a) 0·1789 (b) 0·4164 (c) 0·430
5. (a) 0·9952 (b) 0·1636
6. (a) 0·8423 (b) 0·7134
7. 0·8
8. (a) 0·0355 (b) 0·9790 (c) 0·0565
9. (a) 0·1937 (b) 0·9298 (c) 0·0702
10. (a) 0·1285 (b) 0·8159 (c) 0·0556
11. (a) 0·175 (b) 0·002 (c) 0·003 (d) 0·762
12. (a) 0·1780 (b) 0·0002 (c) 0·1318 (d) 0·1694
13. 0·0956
14. (a) $\frac{1}{6}$ (b) 0·8385 (c) 0·7752 (d) 0·2248
15. 0·1035
16. Joel
17. 7
18. At least 9 attempts.
19. Compare your comments with those of others in your class.
20. Compare your reasoning with that of others in your class.

Miscellaneous Exercise Nine. Page 197.

1. (a) ~35°C (b) ~21°C
2. $2x + 5$
3. $6(2x + 1)^2$
4. $4(2x - 3)$
5. $\frac{1}{(x + 1)^2}$
6. $48x^3 + 18x^2 - 10$
7. $2(2 - 3x)^2(1 - 6x)$
8. $2(4 + 7x)^3(35x + 18)$
9. e^x
10. $10x + e^x$
11. $12e^{3x} + 4x^3$
12. $2e^{2x - 4}$
13. $3e^{3x + 1}$
14. $xe^x(x + 2)$
15. $1 + e^x + xe^x$
16. $\cos x$
17. $-3 \sin 3x$
18. $3 \cos (3x - 5)$
19. $2e^{2x}(2 \cos 4x + \sin 4x)$
20. 0·000752 (correct to six decimal places).
21. On each roll there can be considered to be two outcomes, that of guessing correctly (success) and that of guessing incorrectly (failure). Each time the probability of success remains the same (1/6). Thus we have repeated, independent, Bernoulli trials and we are considering the number of successes. Hence a binomial distribution would be an appropriate model.
22. Answers to be checked with calculator.
23. (a) 54 (b) 0 (c) $12(2x - 1)^2$ (d) 300
24. $6 + 2e^2$
25. Answers to be checked with calculator.
26. $-\frac{9}{(x - 1)^2}$
27. (a) 0·375 (b) 0·3 (c) 0·25
28. 6
29. $y = x^2 - 5x + 11$
30. $y = 5x^2 - 6x + 1$
31. $f(x) = 6 - 2(8 - 2x)^3$
32. $x = \frac{6}{3t + 1} + 3$

33. (a) $y = \frac{(2x+1)^5}{2} + \frac{7}{2}$ (b) 4 (c) 0·5

34. 8·5 units^2

35. $y = 5x^3 - 7x^2 + 3$

36. (a) \$67·65/year (b) \$109·33/year (c) \$199·21/year (d) \$1205·13/year

37. 0·65.

38. Using calculus the small change in the function is approximately 1·24.
f(5·04) – f(5) = 1·2448. The value of 1·24 is a good approximation.

39. V changes by approximately 9%.

40. 1 units^2

41. –5, –7·125

42. \$606

43. Velocity of object at time t is $-\sin t \times e^{\cos t}$ m/sec.
When $t = \pi/2$, velocity = –1 m/sec.

44. (a) $\frac{1}{4}$ (b) $\frac{17}{3}$

45. $\frac{2\cos x}{(1-\sin x)^2}$

46. (a) Zero. (P(Zero sixes) = 0·482.)

(b) More likely to get a number other than zero sixes than to get zero sixes.
P(zero sixes) = 0·482, P(not zero sixes) = 0·518.

(c) Long term average number of sixes would be close to $\frac{2}{3}$.

47. $\frac{-3(\pi\sqrt{3}+3)}{2\pi^2}$

48. Maximum turning point at $\left(-\frac{5\pi}{3}, 2\right)$. Minimum turning point at $\left(-\frac{2\pi}{3}, -2\right)$.
Maximum turning point at $\left(\frac{\pi}{3}, 2\right)$. Minimum turning point at $\left(\frac{4\pi}{3}, -2\right)$.

49. 0·5(n + 1)

50. (a) a = –5·5, b = 4, c = 6·5 (b) (–7, 10·5)

51. $\frac{11}{4\pi}$ units^2

52. (a) $\frac{dx}{d\theta} = 10\cos^2\theta - 10\sin^2\theta \quad (= 10 - 20\sin^2\theta)$

(b) (Justify *maximum* either by considering the sign of $\frac{dx}{d\theta}$ either side of 45° or by consideration of second derivative.) $x_{max} = 5$.

(c) $x = 5\sin 2\theta$
Maximum value of sin A is 1, when angle A is $\pi/2$ radians, or 90°.
Thus maximum value of 5 sin 2θ is 5, when 2θ is $\pi/2$. This maximum x value is 5 when angle of projection is $\pi/4$ radians, or 45°, thus confirming part (b) answers.